

# Additional Topics In Math

Students will solve problems associated with length, area, volume, and scale factors using geometric figures; determine congruence, similarity, and sufficiency using concepts and theorems about vertical angles, triangles, and parallel lines cut by a transversal; solve problems using the Pythagorean theorem, right triangle and unit circle trigonometry, and properties of special right triangles; and use properties and theorems relating to circles to solve problems, all from high school courses preparatory for the math aligned with college and career readiness expectations. This Math Section Content Domain represents  $\approx 15\%$ / 5–7 questions

## **Skill/Knowledge Testing Points**

Area and volume Lines, angles, and triangles Right triangles and trigonometry Circles

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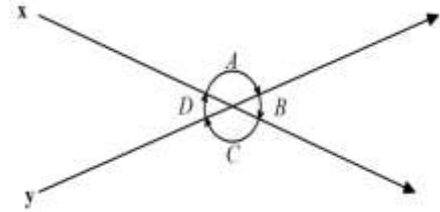
# LINES AND ANGLES

**SUPPLEMENTARY ANGLES:** If the sum of two angles is  $180^\circ$  then the angles are called supplementary angles.

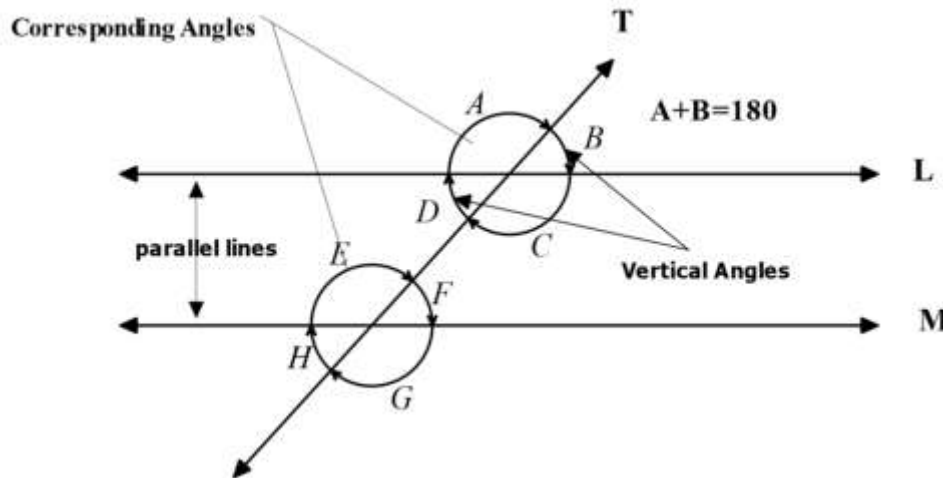
**COMPLEMENTARY ANGLES:** If the sum of two angles is  $90^\circ$  then the two angles are called complementary angles.

**VERTICALLY OPPOSITE ANGLES:** When two lines intersect, the angles formed opposite to each other at the point of intersection are called vertically opposite angles.

In the figure to the right  $x$  and  $y$  are intersecting lines, angle  $A$  and angle  $C$  are one pair of vertically opposite angles.



## PARALLEL LINES



Here,  $L$  and  $M$  are two parallel lines, intersected by a line  $T$ . The line  $T$  is called a Transversal

When a transversal intersects two parallel lines,

1. The corresponding angles are equal.  $\angle A = \angle E$ ,  $\angle B = \angle F$ ,  $\angle C = \angle G$ , and  $\angle D = \angle H$ .
2. The vertically opposite angles are equal.  $\angle A = \angle C$ ,  $\angle B = \angle D$ ,  $\angle E = \angle G$ , and  $\angle F = \angle H$ .
3. The alternate interior angles are equal.  $\angle D = \angle F$ , and  $\angle C = \angle E$ .
4. The alternate exterior angles are equal.  $\angle A = \angle G$ , and  $\angle B = \angle H$
5. The pair of interior angles on the same side of the transversal is supplementary. i.e. angles  $C$  and  $F$  are Supplementary

We can say that the lines are parallel if we can verify at least one of the aforementioned conditions.

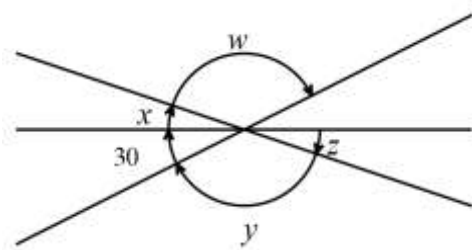
**PERPENDICULAR LINES:**

When there is a right angle between two lines, the lines are said to be perpendicular to each other.

**Theorem 1.** If two lines are parallel and a third line is perpendicular to one of the parallel lines, it is also perpendicular to the other parallel line.

**Theorem 2.** If two lines are perpendicular to the same line, they are parallel to each other

**EXAMPLE 1**



In the figure above,  $w+x+y+z =$

- A) 330
- B) 300
- C) 270
- D) 240

**Solution**

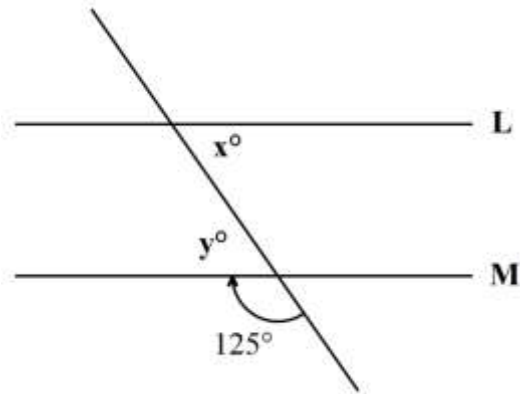
The angle that is  $30^\circ$  has a vertically opposite angle.

$$30^\circ + 30^\circ + w + x + y + z = 360^\circ$$

$$\therefore w + x + y + z = 300$$

.....Ans ( B )

**EXAMPLE 2**



In the figure above, if L is parallel to M, then what is the value of  $x+y$ ?

- A) 150
- B) 125
- C) 120
- D) 110

**Solution**

The angles represented by  $y^\circ$  and  $105^\circ$  are supplementary because they form a straight line.

$$y + 125 = 180$$

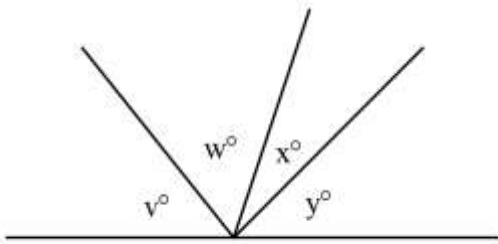
$$y = 55$$

$x$  and  $y$  are alternate interior angles

$$\therefore x = 55$$

$$\text{So } x+y=110 \quad \text{.....Ans(D)}$$

**EXAMPLE 3**



Note: Figure not drawn to scale

In the figure above,  $v=2w$ ,  $w= 2x$ , and  $x = \frac{y}{3}$ .

What is the value of  $y$ ?

- A) 18
- B) 36
- C) 45
- D) 54

**Solution**

Write all the angles in terms of  $y$

$$x = \frac{y}{3}, w = \frac{2y}{3}, v = \frac{4y}{3}$$

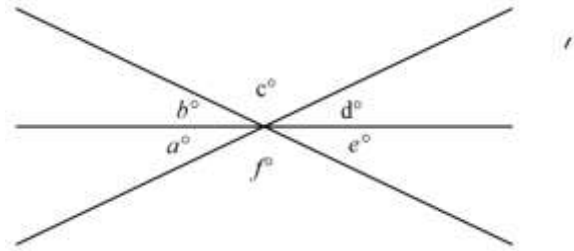
$$\therefore y + \frac{y}{3} + \frac{2y}{3} + \frac{4y}{3} = 180$$

$$\frac{10y}{3} = 180$$

$$y = 54$$

.....Ans (D).

**EXAMPLE 4**



Which of the following must be true of the angles marked in the figure above?

- I.  $a + b = d + e$
- II.  $b + e = c + f$
- III.  $a + c + e = b + d + f$

- A) I only
- B) I and II only
- C) I and III only
- D) I, II, and III

**Solution**

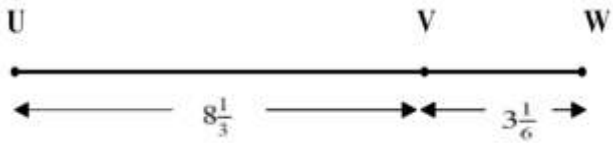
We have three pairs of vertical angles around the point of intersection:  $a$  and  $d$ ,  $b$  and  $e$ , and  $c$  and  $f$ . Therefore  $a = d$ ,  $b = e$ , and  $c = f$ .

- I.  $a + b = d + e$ . Since  $a = d$  and  $b = e$ , this is true
- II. This statement must be true (vertical opposite angles)

Statements I and III must be true

.....Ans (C)

**EXAMPLE 5**



In the figure above, if  $X$  (not shown) is the midpoint of  $UV$  and if  $Y$  (not shown) is the midpoint of  $VW$ , what is the length of  $XY$ ?

- A)  $\frac{23}{2}$
- B)  $\frac{11}{2}$
- C)  $\frac{23}{4}$
- D)  $\frac{19}{3}$

**Solution**

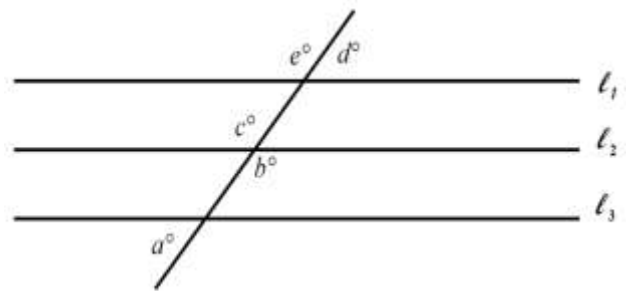
If  $X$  and  $Y$  are the midpoints of  $UV$  and  $VW$ , respectively, then the length of  $XY$  equals half the length of the entire line segment  $UW$ .

$$UW = 8\frac{1}{3} + 3\frac{1}{6} = \frac{23}{2}$$

$$XY = \frac{1}{2} \times \frac{23}{2} = \frac{23}{4}$$

.....Ans (C)

**EXAMPLE 6**



In the figure above  $l_1 \parallel l_2$  and  $l_2 \parallel l_3$ . What is the value of  $a + b + c + d + e$ ?

- A) 270
- B) 360
- C) 450
- D) It cannot be determined from the information given.

**Solution**

$$a + b = 180$$

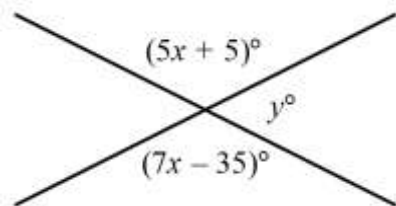
$$c + d = 180$$

We're left with  $360 + e$  since we don't know the value of  $e$ , we cannot determine the sum.

.....Ans (D).

Practice Questions: **LINES AND ANGLES.**

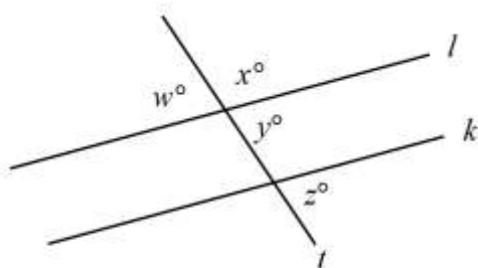
1. ☆☆☆



Two lines intersect as shown. What is the value of  $y$ ?

- A) 40
- B) 70
- C) 75
- D) 80

2. ☆☆☆

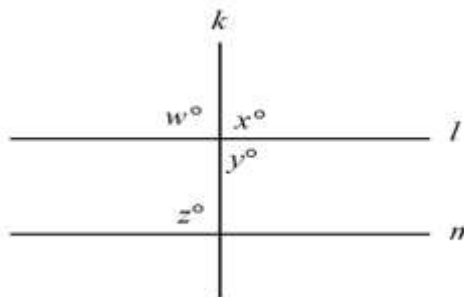


Note : Figure not drawn to scale

In the figure shown, line  $t$  intersects line  $l$  and  $k$ , which additional piece of information is sufficient to prove that lines  $l$  and  $k$  are parallel?

- A)  $x > 90$
- B)  $w < 90$
- C)  $w = y$
- D)  $y = z$

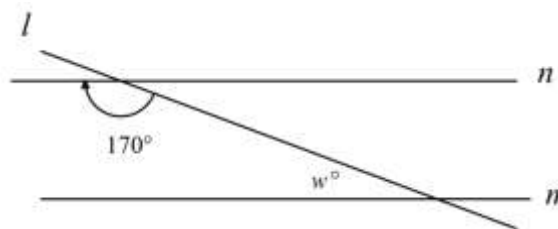
3. ☆☆☆



In the figure shown, line  $l$  and  $m$  each intersect like  $k$ . Which of the following is sufficient to prove that lines  $l$  and  $m$  are parallel?

- A)  $w = y$
- B)  $w = z$
- C)  $x = y$
- D)  $x = z$

4. ☆☆



Note figure not drawn not drawn to scale.

In the figure, line  $m$  is parallel to the line  $n$ . What is the value of  $w$ ?